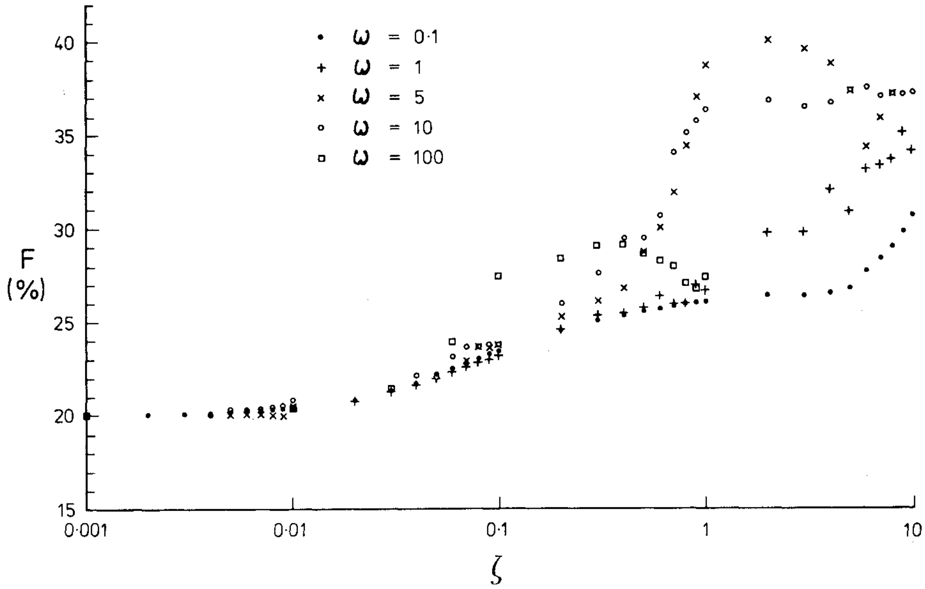

Fig. 2 Variation of percentage peak-to-peak oscillation of centerline velocity (F) with streamwise distance for $\epsilon = 0.1$.

increase sharply for $\zeta > 1$. The accuracy of the numerical technique was extensively evaluated and the authors believe that the behavior of F for $\zeta > 1$ is a correct prediction within the framework of the thin shear layer equations.

Conclusions

The transformation used in solving the thin shear layer equations for the unsteady laminar free jet is successful in reducing substantially the number of grid points in the transverse direction when compared with methods of other workers. It therefore has potential in turbulent jet calculations. Although the unsteady effects, which are important in the prediction of instantaneous quantities, do not influence the mean quantities within the computed frequency range, both theoretical and experimental studies of their influence on the mean flow characteristics at frequencies several orders of magnitude higher than those presented here are warranted.

Acknowledgment

This work was supported by the Australian Research Grants Committee.

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AIAA 81-4142

Triple-Point Trajectory of a Strong Spherical Shock Wave

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Introduction

WHEN a spherical shock encounters a planar or conical wall, a shock transition from regular to Mach reflection takes place depending on its angle of incidence, as shown in Fig. 1. This Note presents an approximate method to predict the triple point trajectory of a strong spherical shock over a planar or conical wall.

Received Sept. 3, 1980; revision received Dec. 15, 1980.

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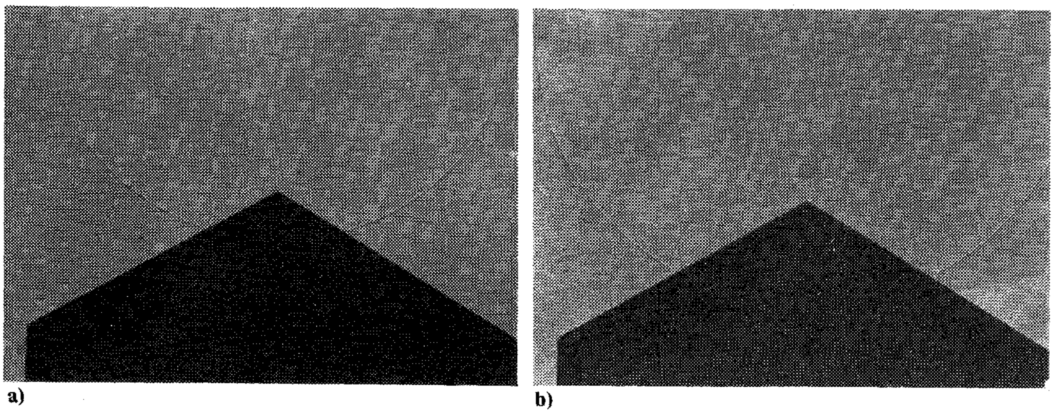

Fig. 1 Direct shadowgraphs of spherical shock diffraction along a cone; $\alpha = 60$ deg, $M_s = 2.94$. a) Regular reflection, b) Mach reflection.

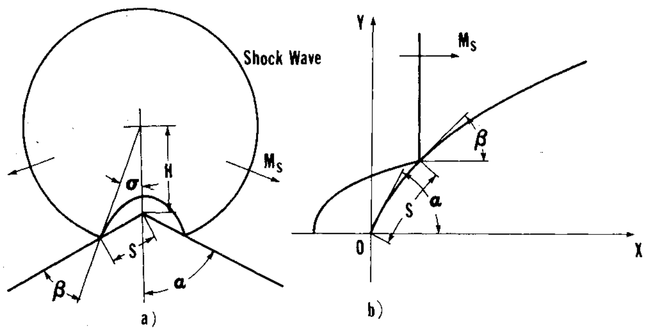

Fig. 2 a) Schematic of spherical shock diffraction along a cone. b) Plane shock diffraction along an equivalent shape wall. Note α, β, and S in Fig. 2a are equal to those in Fig. 2b.

Good agreement is found between the present analysis and the experiments by Dewey et al.,[1] Dipole West Shot No. 8 and conventional shock tube experiments.[2] This method is good provided that the deformation of Mach stem is not significant.

Analysis

Direct shadowgraphs of spherical shock diffraction for $M_s = 2.94$ over a cone of semiapex angle $\alpha = 60$ deg are shown in Fig. 1 a) for regular reflection, and b) for Mach reflection.

The spherical shock diffraction process along a cone, as shown in Fig. 2a, is assumed to be equivalent to a planar shock diffraction process over a curved wall, as shown in Fig. 2b. In Figs. 2a and 2b, the distance S along the body and the inclination angle β are common and equal to each other. Therefore, the following relationships are valid:

$$\frac{dY}{dX} = \tan\beta, \quad \beta = \alpha - \sigma \tag{1a}$$

$$dS = \sqrt{(dX)^2 + (dY)^2} \tag{1b}$$

$$S = \frac{\sin\sigma}{\sin\alpha\sin\beta} \quad \text{or} \quad \frac{dS}{d\beta} = -\frac{1}{\sin^2\beta} \tag{1c}$$

where all the variables having a dimension of length are nondimensionalized with $H\sin\alpha$, and the other notations are shown in Fig. 2.

Integrating Eqs. (1), we can determine the equivalent curved wall shape as follows:

$$Y = \tfrac{1}{2}\log\frac{1+\cos\beta}{1-\cos\beta}\cdot\frac{1-\cos\alpha}{1+\cos\alpha} \tag{2a}$$

$$X = \frac{1}{\sin\beta} - \frac{1}{\sin\alpha} \tag{2b}$$

For $\alpha = 90$ deg, Eqs. (2) give the shape consistent with that obtained by Schultz-Grunow.[3]

For a Mach reflection along any given body shape, its triple point trajectory can be calculated by using Whitham's method.[4] For a strong shock wave, the relationship between the ray tube area ratio A/A_s and the shock Mach number ratio M/M_s is given as

$$A/A_s = (M_s/M)^n \tag{3}$$

where $n = 5.07$ for $M_s \gg 1$, and $\gamma = 1.4$.

If a Mach stem is straight and perpendicular to the curved wall as shown in Fig. 2b, the coordinates of the triple point are expressed as $(X - \lambda\sin\beta,\ Y + \lambda\cos\beta)$, where the triple point height λ is also nondimensionalized with $H\sin\alpha$. Therefore, the ray tube area ratio is given as

$$\frac{A}{A_s} = \frac{2\lambda(Y + 0.5\lambda\cos\beta)}{(Y + \lambda\cos\beta)^2} \tag{4}$$

The condition that the ray and the shock front are orthogonal to each other provides the following approximate relationship,

$$\frac{M_s}{M} = \left[\frac{dX}{dS} - \frac{d(\lambda\sin\beta)}{dS}\right] \Big/ \left[1 + \frac{\lambda}{2}\sin^2\beta\right] \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (3), we finally have the triple point trajectory expressed as following,

$$\frac{d\lambda}{dS} = \cot\beta(1 + \lambda\sin^2\beta) - \frac{1 + (\lambda/2)\sin^2\beta}{\sin\beta}\left\{\frac{2\lambda(Y + (\lambda/2)\cos\beta)}{(Y + \lambda\cos\beta)^2}\right\}^{\frac{1}{n}} \tag{6}$$

The initial condition is for $\alpha > \beta_{crit}$

$$\lambda = 0 \ \text{at} \ S = \frac{\sin(\alpha - \beta_{crit})}{\sin\alpha\sin\beta_{crit}} \tag{7}$$

and for $\alpha < \beta_{crit}$,

$$\lambda = 0 \ \text{at} \ S = 0 \tag{8}$$

Equation (6) is numerically integrated for the given initial condition by means of the 4th order Runge-Kutta-Gill method.

Results and Discussions

In Fig. 3, the analytical result, λ/H vs S/H for $\alpha = 90$ deg, is compared with the experiment by Dewey et al.,[1] Dipole West Shot No. 8 for an ideal surface. Very good agreement is

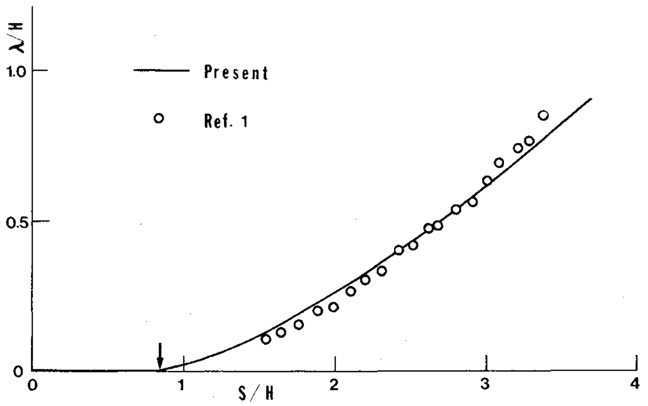

Fig. 3 Triple point trajectory for $\alpha = 90$ deg.

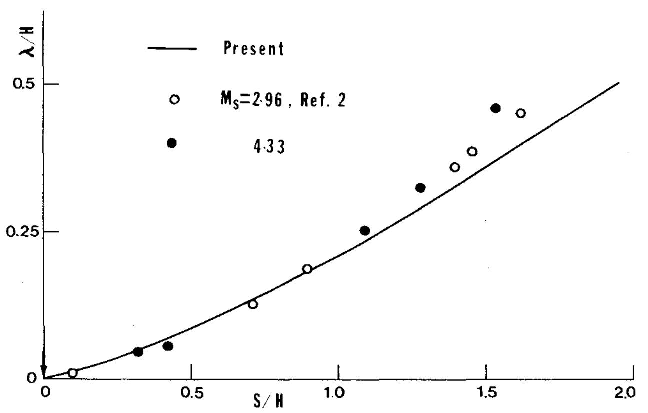

Fig. 4 Triple point trajectory for $\alpha = 30$ deg.

found between the present analysis and the experiment although the shock attenuation effect is ignored in this analysis. In Ref. 1, the spherical shock Mach number was 2.85 at $S/H = 1.36$, which attenuated to $M_s = 1.76$ at $S/H = 3.0$. However, a small discrepancy exists for $S/H > 3.0$. This is because in this analysis the deformation of a Mach stem and the shock attenuation are ignored, whereas in the experiment they become significant for larger S/H. An arrow in Fig. 3 indicates the initiation of Mach reflection. For a strong shock, the critical transition angle β_{crit} over a wedge is about 50 deg for $\gamma = 1.4$ from von Neumann's Detachment Criterion[5]; however, in the present Note this value is experimentally measured to be 44.0 ± 1.0 deg for $M_s > 2.9$ (for details see Ref. 2).

In Fig. 4, the analytical result, λ/H vs S/H for $\alpha = 30$ deg, is compared with the shock tube experiment[2] for $M_s = 2.96$ (data points are shown in open circles) and 4.33 (filled circles). Good agreement is found between the analysis and the experiment. In the shock tube experiment, the center of the spherical shock is slowly moving toward the apex of the cone, that is, the spherical shock is subjected to attenuation. Therefore, the discrepancy between the analysis and the experiment would result from this effect and the other factors previously mentioned.

Acknowledgments

The authors would like to express their thanks to Prof. M. Honda of the Institute of High Speed Mechanics, Tohoku University, for his encouragement throughout the course of this project. Gratitude is also offered to Prof. I. I. Glass of the Institute for Aerospace Studies, University of Toronto, and to Dr. Ben-Dor of Ben-Gurion University of the Negev for their discussions of the present work.

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AIAA 81-4143

Similar Solutions of Unsteady, Laminar Boundary Layers on Swept Cylinders

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Nomenclature

$A_2, A_3, B_1, B_2 B_3$	= constants
C_1, C_2, C_3	= coefficients, Eq. (12)
d	= coefficient, Eq. (12)
F	= dimensionless streamfunction
F''_w	= wall shear-stress parameter in x direction
G	= velocity ratio v/V
G'_w	= wall shear-stress parameter in y direction
K	= constant
Q	= scaling function
t	= time
u, v, w	= velocity components in x, y, and z direction
U, V	= velocity components in x and y direction at the outer edge of the boundary layer
x, y, z	= coordinates
η	= similarity variable
ν	= kinematic viscosity
ψ	= streamfunction
ρ	= density

I. Introduction

SIMILAR solutions of the laminar boundary layer are still of importance as they represent test cases for approximate and numerical approaches. Similar solutions for steady and unsteady two-dimensional flows are well documented in the literature. Steady three-dimensional laminar boundary layers are given by Yohner and Hansen,[1] Hansen and Herzig,[2] and Christian.[3]

The problem of the unsteady laminar boundary layer in the stagnation region of an unswept cylinder was treated by Schuh.[4] Schuh discussed the governing equations, but did not present numerical results. Wuest[5] investigated the unsteady laminar boundary-layer flow in the stagnation region of a swept cylinder. In his work only the velocity component

Received July 11, 1980; revision received Dec. 22, 1980.

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